Poisson upper limits in theory and practice

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Practice

- people do whatever PDG recommends
- people do whatever they consider "traditional"
- people do whatever they have a software package for

How to fix this if we want to?

- perhaps PDG should emphasize that there are many methods...
- make software packages publicly available $\Longrightarrow$ include in CERN libraries?
- many analyses use likelihood/χ^2 fits instead of event counting $\Longrightarrow$ pay more attention to these methods!!!

Rare B searches in CLEO

$$\mathcal{L}(s,b) = \frac{e^{-(s+b)}}{N!} \prod_{i=1}^{N} (s\mathcal{S}_i + b\mathcal{B}_i)$$

$$\mathcal{L}(s) = \int_0^\infty \mathcal{L}(s,b)db$$

$$1 - \alpha = \frac{\int_0^{s_0} \mathcal{L}(s)ds}{\int_0^\infty \mathcal{L}(s)ds}$$

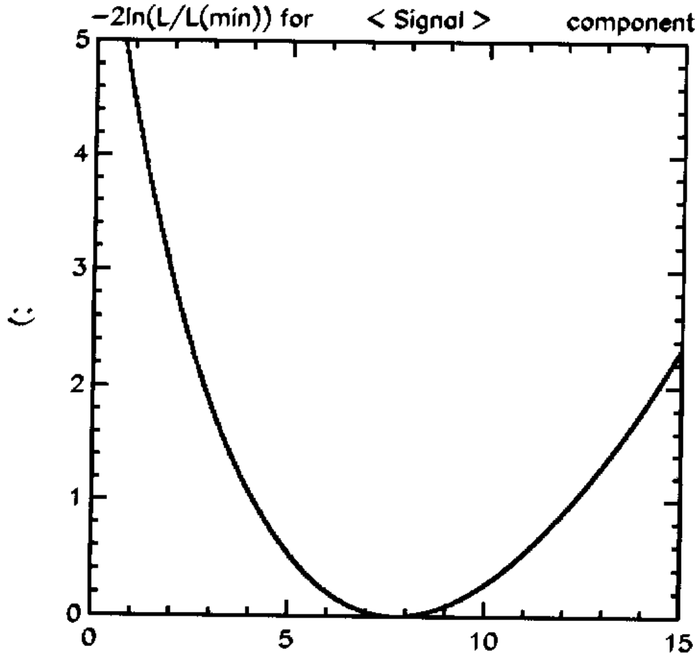


Figure 1: Example of a likelihood fit.

$$\mathcal{L}(s,b) = f(\vec{x}|s,b)$$

$$\mathcal{L}(s) = f(\vec{x}|s)$$

$$\mathcal{L}(s,b) = \frac{f(s,b|\vec{x})f(\vec{x})}{f(s,b)}$$

$$f(s,b) = f(s)f(b)$$

rare B searches (CLEO)	**complete form**
$\mathcal{L}(s) = \int_0^\infty \mathcal{L}(s,b)db$	$\mathcal{L}(s) = \int_0^\infty \mathcal{L}(s,b)f(b)db$
$1-\alpha = \int_0^{s_0} \mathcal{L}(s)ds / \int_0^\infty \mathcal{L}(s)ds$	$1-\alpha = \int_0^{s_0} \mathcal{L}(s)f(s)ds / \int_0^\infty \mathcal{L}(s)f(s)ds$
implicitly uses Bayesian approach with flat prior	

Bayesian Methods

$$\pi(s|n) = \frac{f(n|s)\pi(s)}{\int_0^\infty f(n|s)\pi(s)ds}$$

$$1-\alpha = \int_0^{s_0} \pi(s|n)ds$$

$$f(n|s) = e^{-s}s^n/n! \quad f(n|s) = e^{-(s+b)}(s+b)^n/n!$$

Non-informative Priors

flat	Bayes & Laplace	flat
$1/\sqrt{s}$	Box & Tiao	$1/\sqrt{s+b}$
$1/s$	Jeffreys & Jaynes	$1/(s+b)$

$$\pi(s) \propto \frac{1}{(s+b)^m} \; ; \qquad m = 0,\ 0.5,\ 1$$

$$\boldsymbol{1-\alpha = 1 - \frac{\Gamma(n-m+1, s_0+b)}{\Gamma(n-m+1, b)}}$$

Frequentist Likelihood Method

$$\mathcal{L}(s,b) = \frac{e^{-(s+b)}}{N!}\prod_{i=1}^{N}(s\mathcal{S}_i + b\mathcal{B}_i)$$

Assume confidence interval of the form $(0, s_0)$.

- fit data to extract $\hat{s} = s_{obs}$ and $\hat{b} = b_{obs}$
- assume signal rate s_0
- generate many toy MC experiments

$$f(s) = e^{-s_0}\frac{s_0^s}{s!}; \quad f(b) = e^{-b_{obs}}\frac{b_{obs}^b}{b!}; \quad N = s + b$$

- fit every MC sample to extract $\hat{s}$ and $\hat{b}$
-

$$1 - \alpha = \frac{N_{exp}(\hat{s} > s_{obs})}{N_{exp}(total)}$$

$$1 - \alpha = 1 - \sum_{k=0}^{n} e^{-(s_0+b)}\frac{(s_0+b)^k}{k!}$$

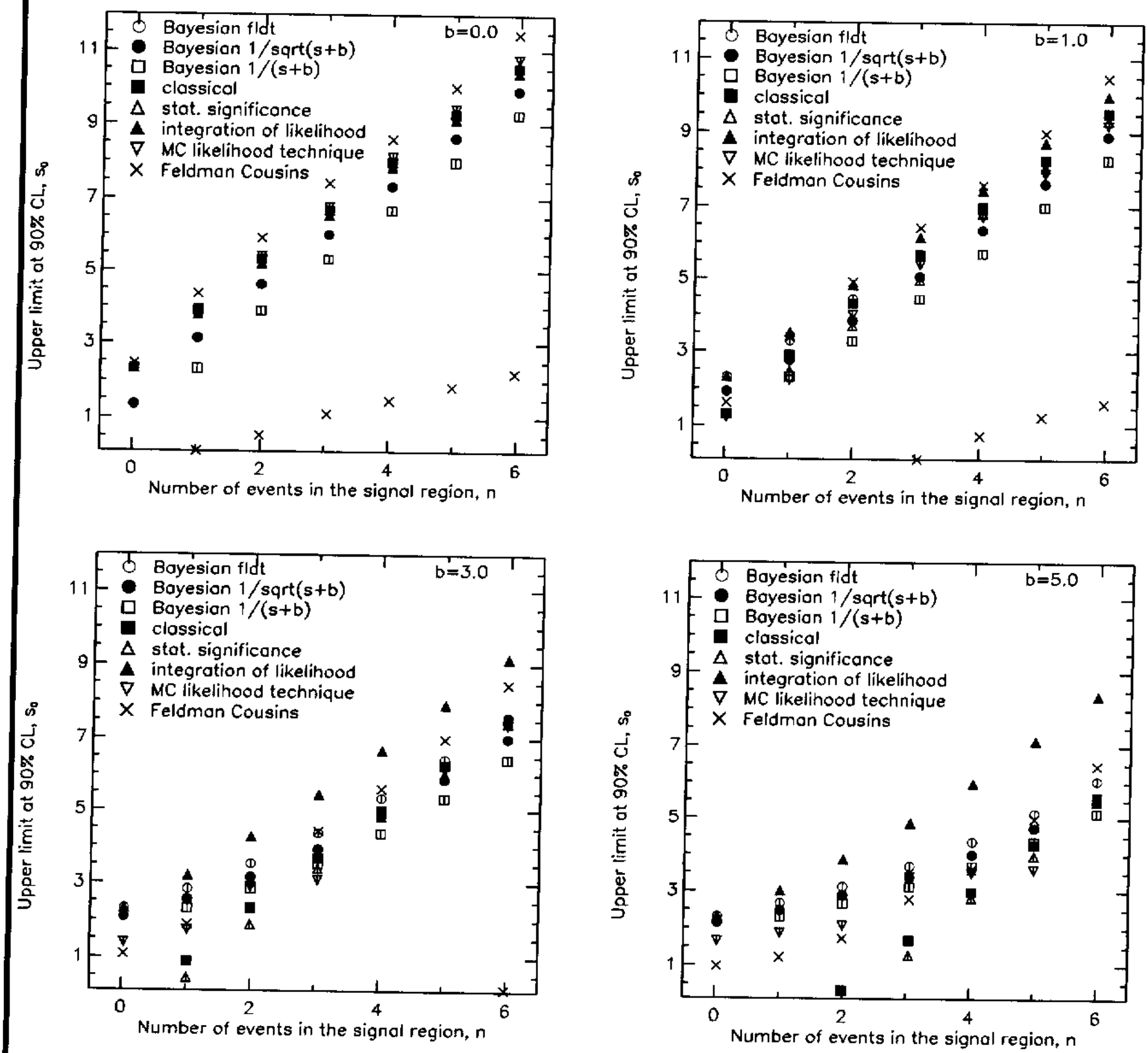
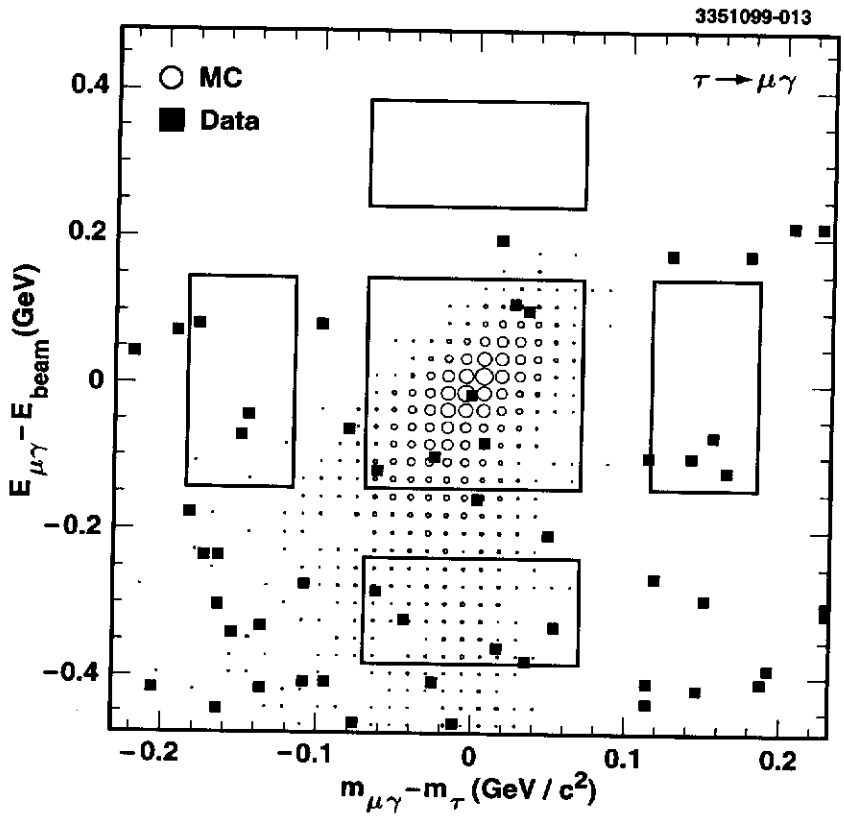


Figure 2: Comparison of various methods.

Search for $\tau \rightarrow \mu\gamma$, CLEO 1999



Method	**UL at 90% CL**
Bayesian with flat prior	**5.76**
Bayesian $1/\sqrt{s+b}$	5.32
Bayesian $1/(s+b)$	4.92
classical	5.03
classical, based on stat. signif.	4.68
integration of likelihood	6.67
Monte Carlo likelihood technique	**3.85**
Feldman & Cousins	~ 6.0